NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3284

EXAMINATION OF THE EXISTING DATA ON THE HEAT TRANSFER OF TURBULENT BOUNDARY LAYERS AT SUPERSONIC SPEEDS FROM THE POINT OF VIEW OF REYNOLDS ANALOGY

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Washington
August 1954

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SUMMARY

Heat-transfer data from four wind-tunnel experiments and two free-flight experiments with turbulent boundary layers have been examined to see whether or not they are well represented by the Reynolds analogy or a modification thereof. The heat-transfer results are put into the form of dimensionless Stanton numbers based on fluid properties at the outer edge of the boundary layer and are compared with skin-friction coefficients for the same Mach numbers and wall to free-stream temperature ratios as obtained from an interpolation of the existing skin-friction data. The effective Reynolds number is taken to be the length Reynolds number measured from the effective turbulent origin, a position which differs importantly from the leading edge of the test surface in some cases.

The data cover the Mach number range from 1.4 to 3.2 at effective Reynolds numbers from 0.4 million to 24 million. They were obtained on a variety of body shapes including flat plates, cones, and pointed slender bodies of revolution. Extremely blunt shapes (such as spheres) were excluded from the correlation. Allowance was made for the differences between cone flow and flat-plate flow but otherwise no corrections for shape were applied.

The results are presented in the form of ratios of experimental Stanton number to skin-friction coefficient and the 32 values obtained all lie within the limits, 0.48 to 0.72. The mean ratio is 0.61 and there is an average deviation of 8 percent from this value. This appears to be an excellent confirmation of the existing theory on this subject.

INTRODUCTION

The Reynolds analogy is an old and widely known method of estimating the heat transfer of a turbulent boundary layer from its skin-friction characteristics, but until very recently, there has not been much experimental evidence to show whether or not it was applicable to turbulent boundary layers at supersonic speeds. In two recent investigations, one by the present author which was not formally published, and one by

C. C. Pappas (ref. 1), the variations with Mach number of heat-transfer data and skin-friction data from various facilities were compared for turbulent boundary layers. It was found that, within the scatter of the experiments, the Mach number variations of the heat transfer and the skin friction were the same, thus suggesting the correctness of the Reynolds analogy or some modification thereof.

A factor which complicates the evaluation of the Reynolds analogy for turbulent flow at supersonic speeds is the dependence of the skin friction and presumably the heat transfer on the ratio of wall temperature to free-stream static temperature. Some recent measurements in the Ames supersonic free-flight wind tunnel, as yet unpublished, have indicated that for a Mach number near 4 a sizable difference exists between the skin friction at large heat transfer and that at zero heat transfer. Furthermore, such differences have also been observed between heated and unheated pipe flows at subsonic speed. No difficulties due to this variation of skin friction with wall temperature ratio were encountered in reference 1 because the data included therein were all for wall temperatures not far removed from the recovery temperature. In fact, a majority of the data now existing for both skin friction and heat transfer are for the condition of small heat transfer. There are available, however, enough data with large heat transfer to permit an estimate of the effects of large heat transfer on both the skin friction and the Reynolds analogy. This estimate must be based on a minimum of data and will be, to that extent, speculative, but the publication of this general framework against which to compare subsequent data should be useful in the development of this field of research.

The purposes of this paper, then, are to generalize the comparisons of skin-friction and heat-transfer data to include those conditions of large heat transfer which will undoubtedly occur in flight; to present these comparisons in a more direct manner than has been used heretofore, as ratios of Stanton number to skin-friction coefficient; and to examine where possible the dependence of the Reynolds analogy (or its modifications) on Mach number, Reynolds number, and wall temperature ratio.

SYMBOLS

a	laminar skin friction constant = $C_F\sqrt{R}$, dimensionless
b,c	stations along turbulent region of cone (Appendix C), ft
d	constant, dimensionless
C_f	local skin-friction coefficient, dimensionless
C_F	average skin-friction coefficient, dimensionless
C_p	heat capacity of air at constant pressure, Btu/slug °F

h	local heat-transfer coefficient, Btu/sec ft^2 °F
$\bar{h}$	average heat-transfer coefficient, Btu/sec ft^2 °F
k	thermal conductivity of air, Btu/sec ft^2 °F/ft
l	total length of test surface, ft
n	constant, dimensionless
Nu	Nusselt number, hx/k, dimensionless
Pr	Prandtl number, $C_p\mu/k$, dimensionless
q	heat-transfer rate, Btu/sec ft^2
r	local radius of cross section of a cone, ft
R	Reynolds number, dimensionless
s	streamwise coordinate along body surface, ft
St	local Stanton number, $h/(C_p\rho u)_1$, dimensionless
$\overline{St}$	average Stanton number, $\bar{h}/(C_p\rho u)_1$, dimensionless
T	temperature, °F abs
T_r	recovery temperature, wall temperature with zero heat transfer, °F abs
u	velocity component parallel to surface, ft/sec
x	coordinate in direction of free stream, ft
y	coordinate normal to test surface, ft
β	half-angle of cone, deg
δ	boundary-layer thickness, ft
θ	boundary-layer momentum thickness, ft
μ	air viscosity, slugs/ft sec
ρ	air density, slugs/ft^3
τ	shearing stress at wall, lb/ft^2

Subscripts

o	undisturbed free-stream conditions
1	outer edge of boundary layer
i	incompressible or zero Mach number case
L	laminar
s	outer edge of sublayer
T	turbulent
w	wall

HISTORY

The analogy between the processes of heat transfer and momentum transfer in a turbulent boundary layer was first suggested by Osborne Reynolds in 1874. As is well known, this hypothesis leads to the following relationship between the dimensionless coefficients of heat transfer and skin friction:

$$St \equiv \frac{h}{C_p \rho u} = \frac{1}{2} C_f \tag{1}$$

In early experiments with air flow in pipes, Reynolds theory appeared to be confirmed but was later shown to overestimate the heat transfer of water in pipes by several hundred percent. The reasons for this discrepancy were developed by Prandtl in 1910 and Taylor in 1916 (refs. 2 and 3). They modified the Reynolds theory to allow for the existence at the base of the turbulent boundary layer of a laminar sublayer controlling the heat transfer and skin friction at the wall. The modified Reynolds analogy equation of Prandtl and Taylor,

$$St = \frac{\frac{1}{2} C_f}{1 - (u_s/u_1)(1 - Pr)} \tag{2}$$

shows a dependence of the Stanton number on the Prandtl number of the fluid and on the velocity ratio, u_s/u_1, at the outer edge of the laminar sublayer. For a Prandtl number of 1, equation (2) reduces to equation (1). For a Prandtl number of 0.72 (air at ordinary temperatures) and for $u_s/u_1 = 0.5$ (a value that appears to be approximately correct for a wide variety of test conditions), equation (2) becomes

$$St = 0.58 C_f$$

Thus, the theory of Prandtl and Taylor shows that equation (1) should be approximately correct for air at ordinary temperatures but in error for fluids having Prandtl numbers much different from 1. Some fluids have Prandtl numbers as large as 700. Although equation (2) is known to become less accurate at such high Prandtl numbers, the effect of the modification is nevertheless to reduce drastically the ratio of St/C_f.

The accuracy of this theory for the low-speed turbulent flow of various fluids was investigated by Colburn, reference 4, who studied the heat-transfer and skin-friction data from a large number of pipe-flow experiments. He correlated the heat-transfer and skin-friction data using the equation,

$$St = \frac{1}{2}\frac{C_f}{Pr^{2/3}} \tag{3}$$

and found that it would satisfactorily represent the data over a wide range of Prandtl and Reynolds numbers. Although this equation is of different form from the theoretical relation of Prandtl and Taylor, the two are, for practical purposes, equivalent (if u_s/u_1 is chosen as 0.5) at Prandtl numbers below 10 as is shown in figure 1.[1]

[1]Colburn's equation has frequently been stated in terms of Nusselt number, a conversion which can be readily made as follows:

$$\frac{h}{C_p \rho u} = \frac{1/2\ C_f}{(C_p \mu/k)^{2/3}}$$

Multiplying both sides by $\dfrac{C_p \rho u x}{k}$ gives

$$\frac{hx}{k} \equiv Nu = \frac{C_f/2}{(C_p\mu/k)^{2/3}}\frac{C_p\mu}{k}\frac{\rho u x}{\mu} = \frac{1}{2} C_f\, Pr^{1/3}\, R$$

Substituting the power law expression for local incompressible turbulent skin friction,

$$C_f = \frac{0.058}{R^{1/5}}$$

gives

$$Nu = 0.029\ Pr^{1/3}\ R^{4/5}$$

This presentation of the equation obscures the Reynolds analogy basis on which it was obtained, and has the further disadvantage that within the Nu, large variations in x overshadow the relatively small variations in h; that is, the change in Nu with x is largely a reflection of the change in x. It is more desirable to nondimensionalize the heat-transfer coefficient with quantities independent of x or slowly changing along x.

Colburn's investigation showed that equation (3) was in agreement with the data for low-speed turbulent flow. It was not a foregone conclusion that the same would be true in air at supersonic speeds because the primary process causing heat transfer at supersonic speeds is the conversion of kinetic energy to heat in the boundary layer, a process which can be neglected in low-speed flow. In a recent theoretical paper, reference 5, Rubesin has investigated the applicability of a modified Reynolds analogy to air at supersonic speeds and has concluded that the ratio of St/C_f should be essentially the same as given by Prandtl and Taylor or Colburn for air in low-speed flow. Specifically, Rubesin found in the Mach number range from 0 to 5, the Reynolds number range from $R_\theta = 10^3$ to 10^6 (corresponding to a very broad range of flat-plate-length Reynolds numbers from less than 1 million up to the order of 1 billion), and the wall temperature range from T_1 to T_r, that the ratio St/C_f would lie within the limits 0.58 to 0.62, depending to that extent on the wall temperature, Reynolds number, and Mach number. The existing experimental evidence relating to the correctness of this theory at supersonic speeds has been reviewed in the introduction.

ANALYSIS OF SKIN-FRICTION AND HEAT-TRANSFER DATA

Since the present purpose is to compare experimental Stanton numbers with experimental skin-friction coefficients, and since, in general, simultaneous measurements of the skin friction and heat transfer have not been made, it is necessary to relate somehow the existing separate data on skin friction and heat transfer. This immediately requires that one or the other set of data be interpolated so as to make possible comparisons at equal conditions of Mach number, wall to free-stream temperature ratio, and Reynolds number. The natural choice is to interpolate the skin-friction data because it is more complete and better correlated at the present time.

Skin-Friction Data

There are three kinds of skin-friction data which will be collected for use in this report. They are: data from supersonic wind tunnels for the condition of small heat transfer; data from heated pipe-flow experiments with large heat transfer at subsonic speeds, and data from free-flight experiments with large heat transfer at high supersonic speeds.

In the first category, the wind-tunnel data at the condition of small heat transfer, there exists an extensive body of experimental results which have been summarized in reference 6. For present use two sets of results will be selected, those of references 6 and 7. These two sets of data were obtained by direct measurement of the skin

friction rather than by any indirect process and are in excellent agreement with one another. They are also in general agreement with but lie somewhat below the many other experiments which have been obtained by indirect methods. The data from these two references are shown in figure 2. They cover the speed range from $M=0.5$ to $M=4.5$.[2]

The free-flight data for the condition of large heat transfer at high supersonic speeds are as yet unpublished. They are measurements made in the Ames supersonic free-flight wind tunnel and are discussed in Appendix A. (Appendix A is not intended as a complete report on these tests but rather is intended to serve as a background to the use of those results in the present report.) As of this moment the data have been restricted to a few test conditions and those shown herein will be for Mach numbers near 4 and 7 with ratios of wall temperature to free-stream static temperature of 1.0 and 1.8, respectively. These data are shown in figure 2 and fall well above the wind-tunnel data for zero heat transfer near $M=4$.

The heated pipe-flow data referred to above are largely the result of chemical and mechanical engineering experience involving fluid flow in heat exchangers. This experience has led to the formulation of a so-called film-temperature rule (given, e.g., in ref. 4) for calculating the skin friction in turbulent pipe flow at high heating rates. This rule specifies that the usual skin-friction equation be used but that the fluid density and viscosity in the Reynolds number and the density in the skin-friction coefficient be evaluated at the film temperature, defined as the average of the pipe wall temperature and the bulk temperature of the fluid or, in present notation, as $\frac{1}{2}(T_W + T_1)$. Rather than to use this rule directly, it was felt that it would be more satisfactory to present a set of data in this paper. There exists a set of results for heated air flow in pipes obtained in NACA tests at the Lewis Laboratory, reference 8. These data are in satisfactory agreement with the film-temperature rule but show a slightly smaller effect of wall temperature ratio on the skin friction than would be predicted from the rule. One point interpolated from the data of reference 8 is shown in figure 2, the point for a wall temperature ratio of 1.8. Additional results from this reference will be introduced in a subsequent paragraph.

The data mentioned above will serve, in the present report, as the basis of an interpolation which leads to values of the skin-friction coefficient at all the test conditions encountered in the heat-transfer experiments. The interpolation proceeds by seeking to define curves of constant wall temperature ratio in figure 2. Two such curves are shown in the figure. They were obtained by connecting the experimental points of equal wall temperature ratio in the three sets of data. The curvature was required to be concave upward at the higher Mach numbers from the condition that C_F/C_{F_i} cannot become zero at any finite Mach number. The shape of the curves was also guided somewhat by the shape of the curve for zero heat transfer. It is clear that the curve for

[2]In figure 2, C_F/C_{F_i} and C_f/C_{f_i} are used interchangeably since, on a flat place, they are equal. Actually, the data of reference 6 are average values and those of reference 7 are local.

$T_w/T_1 = 1.8$ cannot cross the curve for zero heat transfer at more than one point (the point where $T_r = 1.8T_1$). From this condition and the requirement of concavity upwards, it can be deduced that the error due to fairing will certainly be less than 11 percent at M=4.5.

Additional curves for intermediate wall temperature ratios were obtained by first cross-plotting at M=4 the data of figure 2 as shown in figure 3.[a] At selected values of the wall temperature ratio, values of C_F/C_{F_i} from figure 3 were connected to points (of the same wall temperature ratio) on the experimental curve for zero heat transfer and to points based on the data of reference 8 to produce the curves shown in figure 4. This figure, although based on a minimum of data for conditions other than zero heat transfer, is believed by the author to be the best currently available estimate of the dependence of skin friction on Mach number and wall temperature and will be used to evaluate the skin-friction coefficients needed in the present study.

In the use of plots like figure 4, there is implied an assumption that C_F/C_{F_i} does not depend on Reynolds number, or, stated in another way, that the relative variation of C_F with Reynolds number is the same at all Mach numbers and wall temperatures. The validity of this assumption has not been fully proved, but there is some evidence available to support it. Perhaps the best is in reference 6 where it is shown that for zero heat transfer at M=2, C_F/C_{F_i} is independent of Reynolds number for Reynolds numbers between 5 million and 20 million. The assumption will be used in the present paper.

The incompressible skin-friction coefficients, C_{f_i} and C_{F_i}, were derived from the Kármán-Schoenherr equation

$$\frac{0.242}{\sqrt{C_{F_i}}} = \log_{10} C_{F_i} R \tag{4}$$

or its numerical equivalent, for practical purposes, the Prandtl-Schlichting equation,

$$C_{F_i} = \frac{0.46}{(\log_{10} R)^{2.6}} \tag{5}$$

Heat-Transfer Data

The heat-transfer data were reduced to experimental Stanton numbers, based on air properties at the outer edge of the boundary layer, and compared in the form of the ratios, St/C_f, to the skin-friction

[a]It is interesting to note that the requirement of concavity upward at the higher Mach numbers in figure 2 also requires that the curvature in figure 3 be as shown.

coefficients defined by figure 4 and equation (4). Following reference 1, comparisons also were made of the experimental Stanton numbers with the Stanton numbers for incompressible flow as given by the Colburn equation, equation (3). In order to make these comparisons (or to understand the heat-transfer results at all), it was necessary to know in each case the effective length Reynolds number of the flow, which sometimes was appreciably different from that based on distance from the leading edge of the test surface, as will now be explained.

When natural transition occurs at a distance from the leading edge, Reynolds numbers based on distance from the leading edge are relatively meaningless for the turbulent part of the boundary layer since, at any given turbulent station, the boundary layer is thinner and the skin friction and heat transfer are greater than when the flow is fully turbulent from the leading edge. A more appropriate length on which to base the Reynolds number is the distance from the measuring station to the hypothetical turbulent origin, defined as the station from which a fully turbulent flow must originate to duplicate the turbulent part of the test flow. The position of the turbulent origin can be estimated if the position of boundary-layer transition is known. The method that was used herein to estimate the position of the turbulent origin is described in Appendix B.

A further complication in the selection of the effective Reynolds number is due to the fact that frequently, when no boundary-layer trip is used, transition occurs over a region rather than at a fixed point. Detailed examination of the transition region with high-frequency instruments, such as hot-wire anemometers or short-duration shadowgraphs, has shown that transition has a sharply defined location at any instant but that this location is a rapidly varying function of time. (See ref. 9 and ref. 3, page 265.) The data under study include some cases of unsteady transition and the result is that at a given measuring station the effective Reynolds number and the turbulent heat transfer and skin friction are rapidly varying functions of time. The heat transfer measured is the time average value. For the purposes of correlation, this situation was handled simply and only approximately by assuming transition to be fixed at the middle of the transition region.

An additional correction to the effective Reynolds number was applied, in the two cases where the test models were cones, to account for the difference between conical and flat-plate flow. According to Van Driest, reference 10, the local skin friction on a cone with fully turbulent boundary layer is the same as that on a flat plate at one half the Reynolds number based on slant length of the cone and properties at the outer edge of the boundary layer. This rule was used in the computation of skin-friction coefficient and low-speed Stanton number for cones with fully turbulent flow. For cones with transition well back, the Van Driest rule does not apply, so, for a case where this occurred, the method given in Appendix C was developed and applied. (Since this type of flow occurs frequently in practice, the equations of Appendix C may be generally useful.)

Additional modifications, such as changing reference properties to those at the boundary-layer edge, or changing from Nusselt to Stanton number, require little comment or explanation except to note that in the absence of complete information, it was sometimes difficult to find values for the needed reference properties, and the information had to be obtained in devious ways, as, for example, taking the ratio of q/h to find $T_r - T_w$. In the following paragraphs, the individual experiments will be discussed in roughly chronological order. Although a complete description of each one cannot be given, an effort has been made to indicate the experimental setup and how the data were obtained. In addition, the modifications that have been applied to the data are specifically described.

Reference 11.- Scherrer, Wimbrow, and Gowen investigated the local heat transfer from an electrically heated cone of 10° half-angle at a free-stream Mach number of 1.53 in the Ames 1- by 3-foot supersonic wind tunnel no. 1. Local heat-transfer rates were measured by noting the electrical power required in small local segments of the cone wall to hold equilibrium temperatures greater than the air-stream recovery temperature. Although the investigation was largely concerned with laminar flow, a few measurements were made with a lampblack roughness band along the first quarter of the cone length to induce turbulence. Liquid film tests showed the boundary layers then to be completely turbulent.

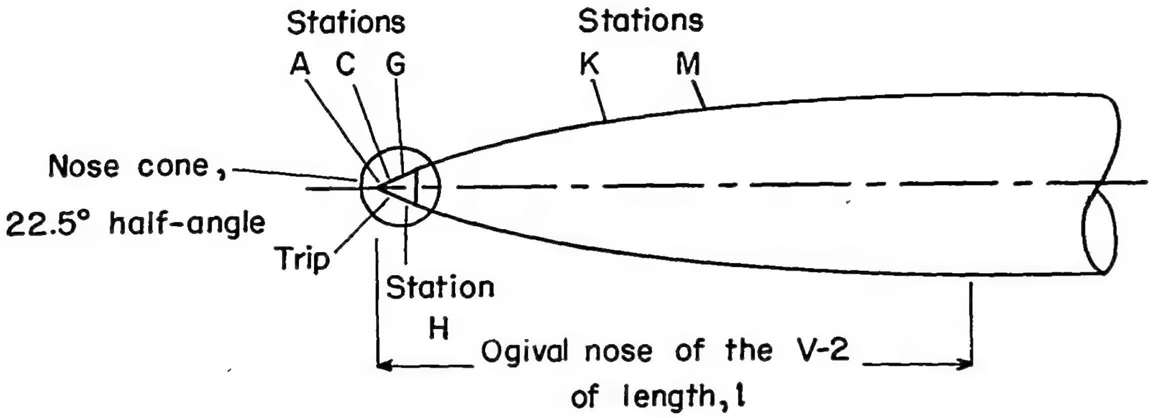
Six out of the thirty-eight data points in the original report were selected for present use. The points chosen agreed with the main body of the data and fell both above and below the mean line through the data drawn by the original authors. All the test conditions of the investigation and four out of the eight measuring stations along the cone are represented by the six points chosen. The data were given in the original report as Nusselt numbers and were converted to Stanton numbers using the following relation between the dimensionless numbers of the flow:

$$St = \frac{Nu}{R(Pr)} \tag{6}$$

Although the boundary layers were reported to be fully turbulent, there was no assurance given that the boundary layers were not thickened by the trip thus causing the turbulent origin to be ahead of the model. This possibility was investigated by plotting the complete data of the investigation in the form Stanton number against Reynolds number and comparing the mean Reynolds number variation with the expected 1/5-power variation. The correspondence was very good and it appeared that the errors in effective Reynolds number due to assuming the turbulent origin to be at the tip were at most 0.2 million and would cause, in the worst case included here, a 4-percent error in the estimated values of C_f and St_i. Therefore, the effective Reynolds numbers were computed by assuming

the turbulent origin to be at the model tip and using the Van Driest rule to define the equivalent flat-plate flow. The converted data are listed in table I along with the test conditions, the computed skin-friction coefficients, and the low-speed Stanton numbers.

Reference 12.- The rate of rise of skin temperature was measured by Fischer and Norris at six points on a V-2 missile in free flight, three points in line on the polished nose cone with no boundary-layer trip, one point (station H) on the opposite side of the nose cone behind a transverse ridge acting as a trip, and two points about half way back along the ogival nose, presumably on the side having no boundary-layer trip.



Analysis of the heat-transfer data obtained from this flight is complicated by the fact that at every measuring station, the flow was laminar at certain times in the flight, transitional at other times, and fully turbulent at times. This is shown by the heat-transfer data (fig. 7 of the reference report) which, considered as a whole, define a laminar curve and a turbulent curve, with each measuring station showing a separate transition line between the two curves. The time at which each measuring station departed from fully laminar flow or became fully turbulent was read from this figure and this information has been used, in the present analysis, to define the boundary-layer-transition history of the flight, shown as figure 5. In this figure, the over-all test Reynolds number of the missile nose, which varied with time, has been chosen as independent variable, and the distance along the surface from the model tip at which the boundary-layer flow changed in character is the ordinate. The positions of the measuring stations are indicated on the figure and each one except K yielded two boundary points, one between laminar and transitional flow, and the second between transitional and turbulent flow. The data from station K stopped short of reaching fully laminar flow. The data from the five stations are reasonably consistent in that smooth curves can be faired through them to define for every instant of the flight the fully laminar, transitional, and fully turbulent regions of the missile nose. At the lowest test Reynolds numbers,

this figure shows that about half the missile nose was fully laminar and no turbulent flow occurred in the measuring region. As the test Reynolds number was increased, the laminar region shrank and there occurred an extensive transitional region, probably (as indicated from small-scale flight tests) a region of intermittently laminar and turbulent flow. At the highest test Reynolds numbers, the flow just did become turbulent at the forward measuring stations A, C, and G, and thus became, apparently, a fully turbulent flow over the entire nose.

From figure 5, it appeared that the most reliable data on turbulent flow, with the least uncertainty in effective turbulent origin, would be for times earlier than 42 seconds and for the rearmost measuring stations. Accordingly, the data selected for this correlation were the station M data for times between 30 and 42 seconds which were reduced assuming fully turbulent flow with the turbulent origin at the tip. Although no attention was given the data involving extensive transitional flow, it is interesting to note that, qualitatively at least, the data respond as would be expected to the changes in transition point, Reynolds number, and Mach number and that some of the strange quirks and turns in the heat-transfer data, which at first suggest poor measuring accuracy, actually imply good response to the changing test conditions.

The Reynolds numbers at station M were evaluated in terms of air properties outside the boundary layer at the measuring station and the length of run, s. An approximate allowance was made for the lateral spreading of the boundary layer on the ogive using the Van Driest rule for conical flow. No allowance was made for the effects of the Mach number and pressure variations along the surface. The Stanton numbers, which in this case were easily calculated from the data of the original paper, and the other pertinent data for station M are listed in table I.

In addition to the data from station M, data from cone station H, which was preceded by a boundary-layer trip, have been reduced for the present correlation. That the boundary-layer trip was effective in producing turbulent flow at station H is shown by the fact that 13 of the 15 heat-transfer measurements made at this station, namely those for Reynolds numbers, $\rho_1 u_1 s/\mu_1$, greater than 0.6 million, lay along the turbulent heat-transfer curve. At the same time, the flow at station G, the corresponding station on the other side of the body, was laminar or transitional showing that the trip was responsible for the turbulent condition at station H. Therefore, station H data for Reynolds numbers appreciably greater than 0.6 million, specifically from 1.1 million to 5.2 million, were analyzed on the assumption of transition at the trip. The data reduced were for times of 40, 46, 50, and 56 seconds after launching, which will serve to identify them in the original report.

The boundary-layer trip was located one third of the distance from the cone tip to station H. With transition at this location, the Van Driest correction for cones with fully turbulent flow is not applicable.

In order to provide an estimate of C_f and St_i at the measuring station for this condition, the method developed in Appendix C was applied. For the case of fully turbulent cone flow, this method is in satisfactory agreement with the Van Driest rule. As an illustration of its use, C_f at station H was computed as a function of transition point for a fixed Reynolds number, $\rho_1 u_1 s/\mu_1$, of 5.2 million and is plotted in figure 6. As the transition point moves from the tip toward the measuring station, the local skin friction increases, slowly at first, but finally to a value which is 1.44 times the value for transition at the tip in this particular case. For transition behind the measuring station, C_f drops, of course, to the laminar value of Hantzsche and Wendt (ref. 13). The C_f required in the present correlation is that for transition at $s = 4$ inches, and turns out to be only a little higher than for transition at the tip.

In all other respects, the reduction of the data at station H was straightforward, and the results are given in table I.

References 14 and 15.- Monaghan and Cooke measured the heat transfer from a hot plate, maintained by a steam jacket at 212° F, to a colder air stream. The plate was a part of the side wall of a 5- by 5-inch wind tunnel, but was elevated somewhat above the surrounding wall, and the tunnel boundary layer on the test wall was pumped off through a slot just ahead of the plate. No boundary-layer trip was used. The boundary-layer transition was investigated by use of a surface pitot probe translated along the center line of the plate. The boundary layer was found to be laminar near the leading edge of the plate, but transition always occurred within the first one third of the plate length. Specifically, it varied in mean position from $x/l = 0.11$ to $x/l = 0.25$ depending on heat-transfer rate and Mach number. The transition region was appreciable in length, ranging from 0.06 to 0.19 of the plate length, again depending on heat-transfer rate. The transition situation was further complicated by two factors: transverse contamination of the test-plate boundary layer by the tunnel side-wall boundary layer, resulting in tongue-shaped laminar regions; and immersion of the test plate in the side-wall boundary layer to the extent of about 0.03 of the plate width on either side.

The average heat transfer between the entire plate and the air stream was measured by noting the heat input required to hold the plate temperature level at 212° F. The heat transfer as measured is a mixed laminar-turbulent result with the exact division depending on the position and fluctuations in position of the transition point. For use here, the data obtained at four test conditions were corrected for the laminar region by estimating the relatively small laminar heat transfer from theory (ref. 16) and subtracting it from the total heat transfer. (The laminar regions deduced from the transition data ranged from 11 percent to 23 percent of the plate surface in the four cases included here.) The comparison coefficients, St_i and C_F, were estimated for the turbulent portion of the plate, with due allowance for the starting length by the method of Appendix B. The resulting data are given in table I.

Reference 17.- W. B. Fallis at the Institute of Aerophysics, University of Toronto, measured the rate of fall of the temperature of an insulated copper disk, flush-mounted in a copper flat plate in a wind tunnel at a free-stream Mach number near 2.5. The plate and disk were preheated before the test to a temperature about 70° F above the air-stream recovery temperature. The temperatures measured during the test were differentiated with respect to time to give a measure of the heat-transfer rate.

The boundary layer was not tripped and turbulent flow did not begin until near the midchord of the plate. Transition position was investigated by three methods: with an evaporating liquid film, with schlieren pictures, and from the heat-transfer data, all of which gave roughly the same answer. The evaporating films showed a large variation in transition point across the span of the plate. At the plate center line, where the heat-transfer measurements were made, the heat-transfer data gave the most precise indication of transition, showing a sharp rise in heat-transfer rate between stations $x = 4.75$ inches and $x = 5.75$ inches. The data were reduced on the assumption of transition at $x = 5$ inches and allowance for the starting length according to Appendix B gave a turbulent origin at $x = 3.4$ inches.

Data obtained at the four measuring stations behind the transition point were converted from Nusselt numbers, with reference properties evaluated at the recovery temperature, to Stanton numbers, referred to air properties at the boundary-layer edge, and are listed in table I.

Reference 1.- Pappas, in an experiment directly intended to investigate the validity of Reynolds analogy for turbulent flow, measured the heat transfer from an electrically heated flat plate to air streams of Mach number 1.7 and 2.3 in the NACA Ames 6-inch heat-transfer wind tunnel. The heaters were placed under the plate alined normal to the air flow so as to give local values of heat-transfer rate and permit local control of plate temperature. The boundary layers were tripped and the effective turbulent origin was investigated from momentum thickness data. A large number of measurements of the Stanton number were made at Reynolds numbers between 1 million and 10 million, and points included in the present report were taken from the mean line of the data. It was necessary only to read the Stanton numbers and Reynolds numbers from the curves of the original paper, and to calculate the skin-friction coefficients and low-speed Stanton numbers by methods previously described. Values selected were at Reynolds number intervals of 2 million, and are given in table I.

Some new data for $T_W/T_1 = 1$ at $M=3.2$.- Because of the comparative lack of heat-transfer data for the condition of large heat transfer at supersonic speeds, it was decided to include herein some first results of some experiments still in progress. The author together with Barbara J. Short has, in the Ames supersonic free-flight wind tunnel, made some indirect measurements of the heat transfer at a wall temperature

ratio of 1.0 and a Mach number of 3.2 using a Mach-Zehnder interferometer. Instantaneous density profiles in the turbulent boundary layer near the base of a fin-stabilized ogive cylinder having a nose fineness ratio of 5 and an over-all fineness ratio of 30 were computed from interferograms of the flow about the models in free flight. The density profiles were found to be time dependent. The mean profile was assumed to represent the average condition of heat transfer and skin friction. The mean density profile was converted to a mean temperature profile assuming constant static pressure in the boundary layer. The temperature profile data could not be used directly to determine the heat-transfer rate because of large changes in the slope, $\partial T/\partial y$, near the wall and consequent lack of definition of the quantity, $(\partial T/\partial y)_w$. Instead, an energy balance calculation was used to estimate the average heat transfer ahead of the measuring station. The kinetic energy lost by the model due to skin friction was equated to the sum of the kinetic and thermal energies gained by the boundary layer and the heat energy transferred to the body.

The boundary layers were tripped by sandblasting the model tips, and the turbulent origins were estimated by measuring the boundary-layer thickness from the interferograms and shadowgraph pictures and fitting the measurements to a law of the form,

$$\frac{\delta}{x} = \frac{d}{R^n}$$

where d and n are constants. Turbulent origins near but just ahead of the model tip were obtained. The results of applying these procedures to two test models are given in table I. The tests are being continued with models of larger scale to investigate the reproducibility of the results under differing test conditions.

DISCUSSION OF RESULTS

The data listed in table I are plotted in figure 7 in the form, St/St_i against Mach number, and are compared with the skin-friction curves transposed from figure 4. It appears that the heat-transfer data vary with Mach number and wall temperature in roughly the same way as do the skin-friction curves. However, the presence of experimental error and the simultaneous variation of two independent variables make it hard to digest this information, obscuring the exactness or lack of it with which the St and C_f variations coincide. Also, no direct information as to the ratios of St to C_f is given. For these same data, then, the result of plotting the ratios, St/C_f, against Mach number is shown in figure 8. The 32 points plotted in this figure run from 0.48 to 0.72 and have a mean value of 0.61 with a mean deviation of 8 percent and an rms deviation of 10 percent from this value. The data extend through the low and moderate supersonic speed range, specifically from $M=1.4$ to 3.2, and do not show any dependence on Mach number within the limits of the scatter.

The same data are plotted against effective Reynolds number in figure 9, and cover the range from 0.4 million to 24 million. There appears to be a downward trend with increasing Reynolds number as indicated by the dashed line. This is based mainly on the data of reference 1 and those of station M, reference 12. Such a trend was predicted in reference 5 except that the predicted slope was considerably smaller than that shown on figure 9. However, part of the data supporting this trend in figure 9, namely those of station M, reference 12, were obtained in the presence of pressure, Mach number, and temperature variations in the direction of flow. Therefore, the variation shown may not be altogether due to Reynolds number. This will be an interesting point to watch in future experiments since the trend shown is such as to constitute an important reduction in heat-transfer rate at very high Reynolds numbers.

In figure 10, the data have been plotted against the difference between the adiabatic wall temperature and the actual wall temperature, $T_r - T_w$. Whereas it was shown in figure 4 that the skin friction was appreciably dependent on the air and surface temperatures, figure 10 shows that the ratio St/C_f does not depend on the temperatures within the scatter of the data. This result is at present supported by only two sets of data at large heat flow into the wall and is deserving of further investigation.

In addition to the large streamwise variations of flow properties mentioned in connection with station M, reference 12, appreciable variations of certain kinds occurred in some of the other experiments. In reference 11, wall temperature varied appreciably along s. In the new free-flight data, the pressure, Mach number, and boundary-layer-edge temperature varied over the first quarter of the model length, well ahead of the measuring station. These variations must influence the heat-transfer rates, but the exact extent of this influence is unknown. That it is apparently not severe is suggested by the fact that the data from these tests do not disagree badly with data obtained on flat plates at uniform wall temperature.

CONCLUDING REMARKS

The existing data on heat transfer of a turbulent boundary layer at supersonic speeds, when analyzed from the point of view of the modified Reynolds analogy, appear to support that analogy to a remarkable degree. This is true in spite of the fact that many of the experiments are imperfect in some respect, such as having somewhat indefinite location of the boundary-layer transition and extensive transitional regions, or appreciable variations in wall temperature in the direction of flow. Furthermore, the data were collected on a variety of surfaces including flat plates, cones having up to 45° included angle, the ogival nose of a V-2 missile, and a very long ogive cylinder. The average experimental value of the ratio of Stanton number to skin-friction coefficient is 0.61 and

the data considered have a mean deviation of only 8 percent from this result. This is surprisingly and perhaps fortuitously close to the average value predicted by theory which, according to the curves of reference 5, would be 0.602 for the 32 data points considered.

The skin-friction coefficients used to obtain this result are based on an interpolation of the existing experimental data which seeks to define the effects of both Mach number and wall temperature. A well-defined dependence of C_f/C_{f_i} on Mach number exists for the condition of zero heat transfer, but there is a need for more skin-friction data at large rates of heat transfer.

Further experimental work in this area can be profitably devoted to: extending upward the range of Mach numbers, investigating the reality of a Reynolds number effect on St/C_f indicated in the existing data, increasing the amount of data for large heat flow into the wall, and investigating the effects of pressure and temperature gradients in the direction of flow. Specifically, it has not been demonstrated that the analogy will apply in cases of large pressure and temperature variation along the flow direction, such as that occurring on a sphere.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., June 16, 1954

APPENDIX A

A BRIEF DESCRIPTION OF THE FREE-FLIGHT SKIN-FRICTION EXPERIMENTS

The free-flight skin-friction data of figure 2, which have not as yet been formally reported, are measurements taken in the Ames supersonic free-flight wind tunnel using gun-launched models. The models used were tubular in form, with an outside diameter of 1.43 inch, a wall thickness of 0.030 inch, and symmetrically beveled leading edges. The models were flown with their axes oriented parallel to the air stream with supersonic air flow through them as well as around them. Skin-friction force was thus developed at both the inner and outer surfaces. The model lengths were chosen such that the internal oblique shock-wave configuration did not strike the opposite tube wall within the physical length of the model. In fact, the shock waves intersected the model wake many wall thicknesses behind the model, and therefore did not influence either the skin friction or the total drag in any way. The boundary layers were thin compared to the radius of curvature of the surface and thus should be closely representative of a two-dimensional flow.

The velocity at which these models were launched was 4300 feet per second. When fired at this speed into still air at room temperature and pressure, the resultant test conditions were a free-stream Mach number near 4, a length Reynolds number near 5 million, and a turbulent-boundary-layer recovery temperature of about 1950° R. The models were initially at room temperature and flew for only about 10 milliseconds before completing the test, a time too short for appreciable surface temperature rise to occur even at the large heating rates that prevailed. Therefore, the wall temperature in flight was about 1400° R below the adiabatic wall temperature, and T_W/T_1 was equal to 1.0. When the models were fired upstream through the Mach number 2 air stream of the Ames supersonic free-flight wind tunnel, the resultant Mach number was near 7. At this Mach number, tests were run at two length Reynolds numbers, nominally 5 million and 7.5 million; the wall temperature was about 2500° F below the recovery temperature; and the ratio, T_W/T_1, was 1.8.

The skin friction was determined from the model drag, computed from a time-distance history of the model flight with an estimated accuracy of 2 percent. Actually two drag measurements were used in determining the skin friction, the drag of a test model 2 inches long, and the drag of a tare model 1/2 inch long. The difference in their drags is, to a first order of approximation, a measure of the skin friction of the after part of the test model. This result was corrected for small differences between the test and tare models in geometry, base drag, and test conditions. A number of test and tare models fired at nominally equal conditions were used in all possible combinations to produce a number of separate values of the skin friction. These values have been averaged

to produce the single points shown in figure 2. The Mach number 3.9 result is the mean of 12 values which had an rms deviation from the mean of 1.3 percent. The Mach number 7.25 point represents 26 values having an rms deviation of 9 percent.

The boundary layers were tripped in these experiments to produce turbulent flow. The trips were located close to the leading edges so that turbulent flow began ahead of the base of the tare models. This was ascertained from shadowgraph pictures in which laminar and turbulent flows off the base of the tare models could be clearly distinguished. It could further be determined within 0.1 inch from these pictures where the transition occurred by examining the boundary layers for the first eddies and noting the Mach waves in the flow which exist along the edge of the irregular turbulent boundary layer and are absent in the case of laminar flow. The boundary-layer trips used were selected by trial and error as the least ones which would cause turbulence on or near the trailing edge of the trip. At the lower speed, a shallow thread (0.003 inch deep) was sufficient to produce turbulent flow, but at the higher speed, the boundary layer was harder to trip and two rings raised 0.008 inch above the surface were required. The former trip had no detectable drag and was proved experimentally not to thicken the boundary layer. The latter trip had large drag, appreciable compared to the skin friction, and also had a thickening effect on the boundary layer. The data have not yet been corrected for this thickening and it is expected that the final value of C_F/C_{F_i} at $M=7.25$ will be a little higher than is shown in figure 2.

The skin friction, determined as outlined above, is a fully experimental result, but there remains the question, what Reynolds number range does it represent? This must be decided before the corresponding skin friction for incompressible flow can be computed. The Reynolds number range was determined from the transition point as read from the shadowgraph pictures. With the position of transition known, the turbulent origin was calculated as described in Appendix B and the Reynolds numbers were then measured from this position.

The original tests were run with spin-stabilized models for convenience, but later tests run with aerodynamically stabilized models of similar form to determine the effect of spin showed that no significant effect of spin was present in the data. The angles of attack in flight were in general less than 1°, and corrections to the data were applied for the drag due to lift. At these angles of attack, the boundary layers were still of nearly equal thickness around the circumference of the low fineness ratio models used. That the angle of attack did not influence the final skin-friction results was shown by the fact that models having unequal amplitudes of pitching oscillation gave the same skin friction within the scatter of the data.

APPENDIX B

CALCULATION OF THE EFFECTIVE TURBULENT ORIGIN ON A FLAT PLATE

The effective turbulent origin of a boundary layer having transition at a known distance from the leading edge of a flat plate was calculated from the assumption that the momentum loss in the boundary layer, integrated along y, was continuous across the transition point. This is equivalent to assuming that the boundary-layer momentum thickness, θ, is continuous across the transition point. Expressed mathematically the assumption is that at the transition point:

$$\left[\int_0^\delta \rho u(u_1 - u)dy\right]_L = \left[\int_0^\delta \rho u(u_1 - u)dy\right]_T \tag{B1}$$

No precise definitions need be made of the boundary-layer thicknesses except to require that the integrals be extended far enough to give a good estimate of the boundary-layer momentum decrement. Now, by Newton's law,

$$\left[\int_0^\delta \rho u(u_1 - u)dy\right]_L = C_{F_L}\, x_L\, \frac{1}{2}\, \rho_1 u_1^2 \tag{B2}$$

where x_L is the length of the laminar region and C_{F_L} is its average skin-friction coefficient. Likewise

$$\left[\int_0^\delta \rho u(u_1 - u)dy\right]_T = C_{F_T}\, x_T\, \frac{1}{2}\, \rho_1 u_1^2 \tag{B3}$$

where x_T is the hypothetical length of run of a fully turbulent flow which produces the same momentum loss at the transition point as does the actual laminar flow, and C_{F_T} is the average skin-friction coefficient of the hypothetical turbulent region. By (B1), (B2), and (B3),

$$C_{F_L}\, x_L = C_{F_T}\, x_T \tag{B4}$$

The length of laminar run to the transition point, x_L, is presumed to be known, and C_{F_L} can be obtained from any of several laminar-flow theories which agree well. (See, e.g., ref. 16.) However, to obtain x_T, a second relation between C_{F_T} and x_T is required. This is obtained from the following equation for turbulent skin friction.

$$C_{F_T} = \frac{0.072}{R_T^{1/5}} \frac{C_F}{C_{F_i}} \tag{B5}$$

The factor C_F/C_{F_i} corrects the low-speed equation to the required Mach number and T_W/T_1. Multiplying both sides by x_T gives

$$C_{F_T}\, x_T = 0.072 \frac{C_F}{C_{F_i}} \left(\frac{\mu_1}{\rho_1 u_1}\right)^{1/5} x_T^{4/5} \tag{B6}$$

Applying this relation to equation (B4) yields

$$C_{F_L}\, x_L = 0.072 \frac{C_F}{C_{F_i}} \left(\frac{\mu_1}{\rho_1 u_1}\right)^{1/5} x_T^{4/5} \tag{B7}$$

which can be reduced to the form

$$\frac{x_T}{x_L} = \left(\frac{C_{F_L}}{0.072\, C_F/C_{F_i}}\right)^{5/4} R_L^{1/4} \tag{B8}$$

This is the required equation for starting length, x_T, of the turbulent flow on a flat plate.[1]

[1]Using entirely similar procedures, x_T can be calculated from the more exact Prandtl-Schlichting equation for low-speed turbulent skin friction. However, the resulting equation is transcendental and must be solved by trial and error. Equation (B8) should be accurate enough for most purposes.

APPENDIX C

CALCULATION OF LOCAL TURBULENT SKIN FRICTION ON A CONE HAVING TRANSITION AT A GIVEN DISTANCE FROM THE TIP

On a flat plate with turbulent boundary layer the local skin-friction coefficient can be expressed in terms of the Reynolds number based on boundary-layer momentum thickness, R_θ. A simple relationship of this type results from the use of the power law rules for local and average incompressible skin friction.

$$C_F = \frac{0.072}{R^{1/5}} \frac{C_f}{C_{f_i}} = \frac{2\theta}{x} \tag{C1}$$

$$C_f = 0.8\ C_F = \frac{0.058}{R^{1/5}} \frac{C_f}{C_{f_i}} \tag{C2}$$

The Reynolds number is based on the length of run, x, from the effective turbulent origin. It is desired to eliminate x in favor of θ by solving equations (C1) and (C2) simultaneously. From (C1),

$$x = \left(\frac{\rho_1 u_1}{\mu_1}\right)^{1/4} \left(\frac{\theta}{0.036\ C_f/C_{f_i}}\right)^{5/4} \tag{C3}$$

From (C2)

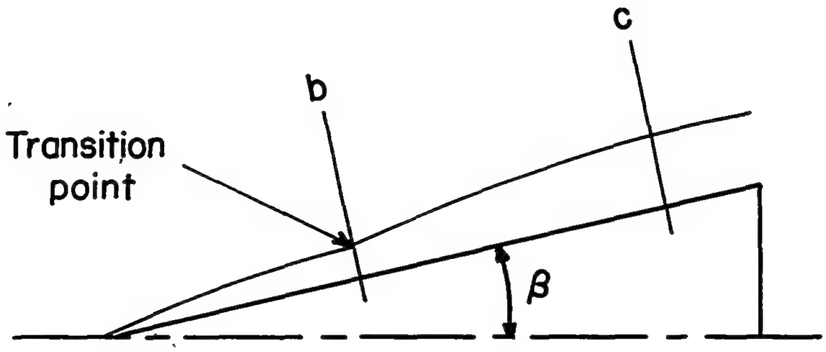
$$x = \left(\frac{0.058}{C_f}\right)^5 \frac{\mu_1}{\rho_1 u_1} \left(\frac{C_f}{C_{f_i}}\right)^5 \tag{C4}$$

Equating the right-hand sides of equations (C3) and (C4) and rearranging gives the desired relation,

$$C_f = \frac{0.025}{{R_\theta}^{1/4}} \left(\frac{C_f}{C_{f_i}}\right)^{5/4} \tag{C5}$$

It will now be assumed that this relation applies also to a cone. Thus, for a cone and flat plate having equal Reynolds number per foot at the edge of the boundary layer and equal momentum thickness, it will be assumed that the local skin friction is the same. However, the boundary layer on the cone will grow in thickness at a slower rate than on the

flat plate due to the lateral spreading of the boundary layer on the cone. This feature of the cone flow will now be mathematically expressed. Consider the flow between stations b and c on a cone, whose half-angle is β.



$$2\pi \int_b^c r\tau ds = \left[2\pi \int_0^\delta (r + y \cos \beta) \rho u(u_1 - u)dy \right]_b^c \tag{C6}$$

Restricting this equation to cases where the boundary layer is thin compared to the body radius,

$$\frac{\delta}{r} \cos \beta << 1 \tag{C7}$$

and using the same definition of θ as is usual in flat-plate flow yields

$$\int_b^c C_f r ds = [2r\theta]_b^c \tag{C8}$$

Taking the differential of (C8) gives

$$C_f r ds = 2d(r\theta) \tag{C9}$$

Equations (C5) and (C9) can be considered as simultaneous equations in C_f and θ, and θ eliminated to give a differential equation for the variation of local turbulent skin friction along a cone having an arbitrary transition point:

$$\frac{dC_f}{ds} + \frac{0.314 \times 10^6}{(C_f/C_{f_i})^5} \frac{\rho_1 u_1}{\mu_1} C_f{}^8 - \frac{C_f}{4s} = 0 \tag{C10}$$

Equation (C10) is integrated between the limits, b and c, by choosing as variable, $1/C_f{}^5$, and using the integrating factor, $s^{5/4}$. This leads to the equation,

$$C_{f_c}{}^{-5} = \left(\frac{b}{c}\right)^{5/4} C_{f_b}{}^{-5} + \frac{6.99 \times 10^5}{(C_f/C_{f_i})^5}\left[R_c - \left(\frac{b}{c}\right)^{5/4} R_b\right] \qquad \text{(C11)}$$

which gives the local skin friction at station c for transition at an arbitrary station, b, provided the turbulent skin-friction coefficient at the transition point, C_{f_b}, is known. The Reynolds numbers, R_b and R_c, are based on boundary-layer-edge properties and the slant lengths, b and c, from the cone tip.

To calculate C_{f_b}, consider the laminar flow terminating at station b. Its momentum thickness is given by the average laminar skin-friction coefficient, C_{F_L}, according to the momentum-thickness equation for cones

$$C_{F_L} = \frac{4\theta_L}{b} \qquad \text{(C12)}$$

Since, by the Hantsche and Wendt rule (ref. 13),

$$C_{F_L} = \frac{2}{3}\sqrt{3}\frac{a}{\sqrt{R_b}} \qquad \text{(C13)}$$

Where a is the laminar constant appropriate to the Mach number at the boundary-layer edge and the wall temperature ratio, then

$$\theta_L = \frac{b}{4}\,\frac{2}{3}\,\sqrt{3}\,\frac{a}{\sqrt{R_b}} \qquad \text{(C14)}$$

Assuming as in Appendix B that at the transition point

$$\theta_L = \theta_T \qquad \text{(C15)}$$

and inserting the value of momentum thickness, given by equation (C14), into equation (C5), yields the local turbulent skin-friction coefficient at the transition point,

$$C_{f_b} = 0.0344\,\frac{(C_f/C_{f_i})^{5/4}}{a^{1/4}\,R_b{}^{1/8}} \qquad \text{(C16)}$$

Equations (C16) and (C11) are sufficient to define C_{f_c} for any specified transition point.

For the case of fully turbulent flow on a cone, $b = o$ and equation (C11) reduces to

$$C_f = \frac{0.068}{R^{1/5}} \frac{C_f}{C_{f_i}} \tag{C17}$$

On a flat plate having the same Mach number at the outer edge of the boundary layer and the same wall temperature ratio,

$$C_f = \frac{0.058}{R^{1/5}} \frac{C_f}{C_{f_i}} \tag{C18}$$

Hence, the local friction on the cone is 0.068/0.058 or 1.17 times that on the plate at stations where the Reynolds numbers based on boundary-layer-edge conditions and actual length of run are the same. The Van Driest rule gives corresponding ratios ranging from 1.14 at $R=10^5$ to 1.08 at $R=10^8$.

The average skin friction on a cone is affected by the fact that at forward stations where C_f is large, there is relatively little wetted area. It has been computed for the case of fully turbulent flow by integrating over the surface the local coefficients defined by equation (C17). The value obtained is 1.047 times the average skin-friction coefficient on the related flat plate.[1]

The equations developed here have been applied to calculating the variation of local skin-friction coefficient at a fixed station on a cone as the transition point varies from the tip up to and past the skin-friction station. The results are shown in figure 6. For fully turbulent flow, the Van Driest and present results are shown to agree satisfactorily. Little change in C_f occurs at first as the transition point moves back. The peak value (given by eq. (C16)) occurs as the transition point reaches the skin-friction station. For transition behind this station, C_f drops to the laminar value of Hantzsche and Wendt.

[1]The reader will note that these ratios of the skin friction on a cone compared to a flat plate are the same as those given in reference 18. The procedures used in this Appendix, while they appear to be quite different from those of reference 18, are actually equivalent except for two differences: (1) The effects of Mach number and wall temperature ratio are included in the present case by use of the ratios, C_f/C_{f_i}, whereas reference 18 makes only a rough allowance for these effects; and (2) The effect of arbitrary transition point is treated more exactly in the present case. The latter aspect of the problem is, of course, the purpose of this Appendix.

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TABLE I.- TABULATION OF THE MODIFIED HEAT-TRANSFER DATA AND THE CORRESPONDING TEST CONDITIONS, AND COMPARISON WITH CALCULATED SKIN-FRICTION COEFFICIENTS

Ref.	M	R, millions	T_w/T_1	St or $\overline{St}$	St_i or $\overline{St_i}$	St/St_i	C_f/C_{f_i}	C_f or C_F	St/C_f or $\overline{St}/C_F$
11	1.41	0.98	1.50	0.00195	0.00226	0.86	0.82	0.00296	0.66
		1.20	1.49	.00182	.00216	.84	.83	.00286	.64
		.76	1.42	.00212	.00238	.89	.83	.00311	.68
		.53	1.54	.00218	.00256	.85	.82	.00327	.67
		.60	1.59	.00210	.00249	.84	.82	.00318	.66
		.73	1.57	.00196	.00241	.81	.82	.00308	.64
12 Sta M	1.38	23.5	1.21	.000973	.00120	.81	.85	.00182	.53
	1.81	24	1.38	.000834	.00120	.69	.78	.00168	.50
	2.10	23	1.51	.000766	.00121	.67	.75	.00161	.48
	2.37	21	1.55	.000766	.00123	.62	.72	.00157	.49
12 Sta H	1.72	2.08	1.25	.00158	.00196	.81	.81	.00254	.62
	2.14	1.50	1.29	.00146	.00209	.70	.77	.00255	.57
	2.37	1.05	1.20	.00149	.00226	.66	.77	.00271	.55
	2.78	.39	1.21	.00179	.00273	.66	.73	.00309	.58
14	2.43	.22 to 3.03	2.78	.00152	.00214	.71	.65	.00218	.70
		.22 to 4.31	3.50	.00126	.00201	.63	.63	.00202	.62
15	2.82	.25 to 2.51	3.40	.00146	.00221	.66	.60	.00206	.71
		.30 to 2.77	3.61	.00144	.00215	.67	.59	.00200	.72
17	2.48	.63	2.41	.00147	.00249	.59	.66	.00256	.58
		.87		.00143	.00234	.61		.00242	.59
		1.13		.00143	.00222	.64		.00231	.62
		1.39		.00133	.00213	.62		.00223	.60
1	1.7	2	1.7(est.)	.00157	.00198	.79	.76	.00240	.65
		4		.00137	.00172	.80	.76	.00214	.64
		6		.00127	.00159	.80	.76	.00202	.63
		8		.00118	.00150	.79	.76	.00193	.61
	2.3	2	2.2(est.)	.00134	.00198	.68	.68	.00215	.62
		4		.00117	.00172	.68	.68	.00192	.61
		6		.00107	.00159	.67	.68	.00180	.59
		8		.00101	.00150	.67	.68	.00173	.58
Unpublished	3.18	0 to 10.6	1.0	.00138	.00185	.75	.75	.00218	.63
		0 to 8.8	1.0	.00127	.00190	.67	.75	.00228	.56

NACA

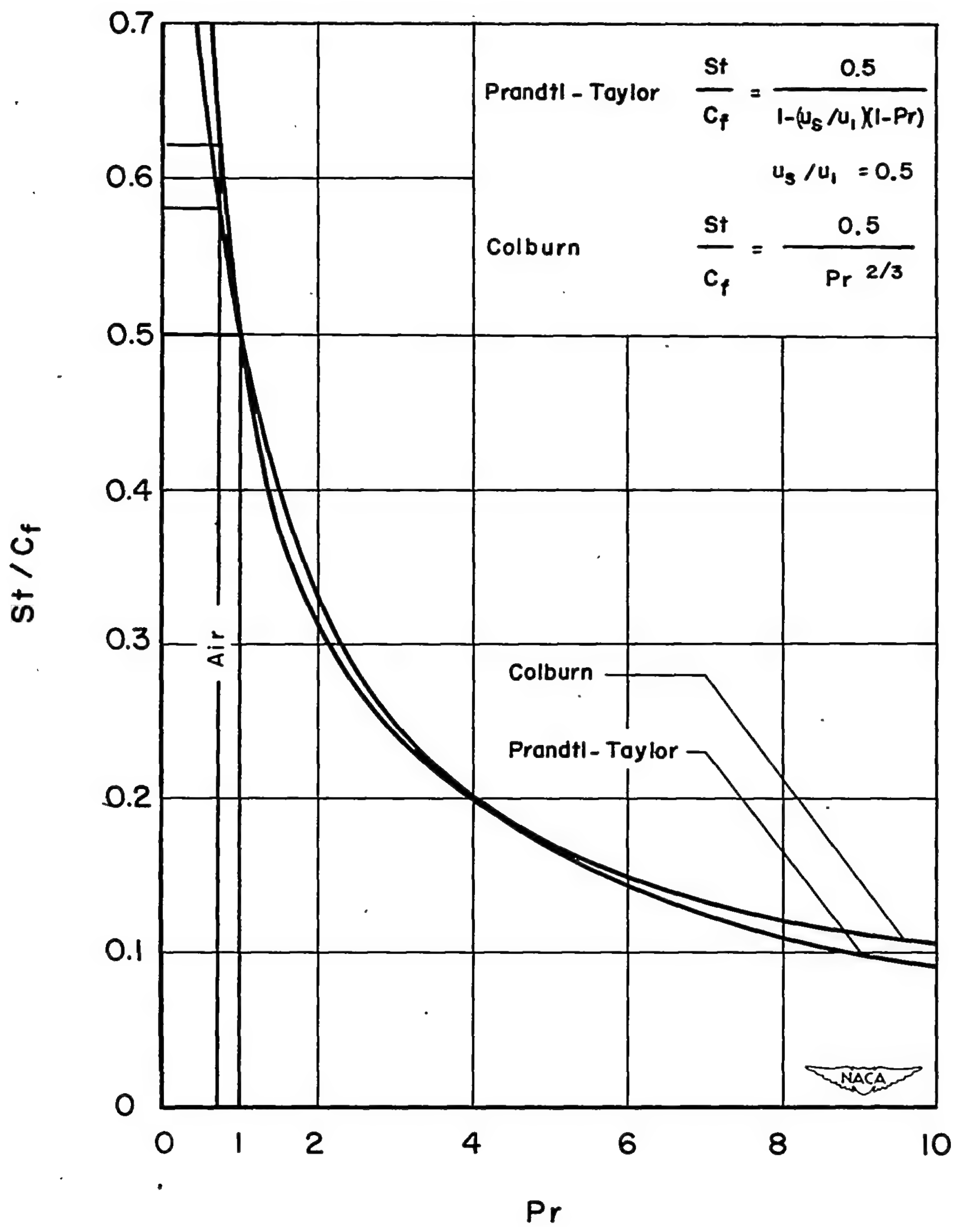

Figure 1. - Comparison of the Prandtl-Taylor and Colburn modifications of Reynolds analogy.

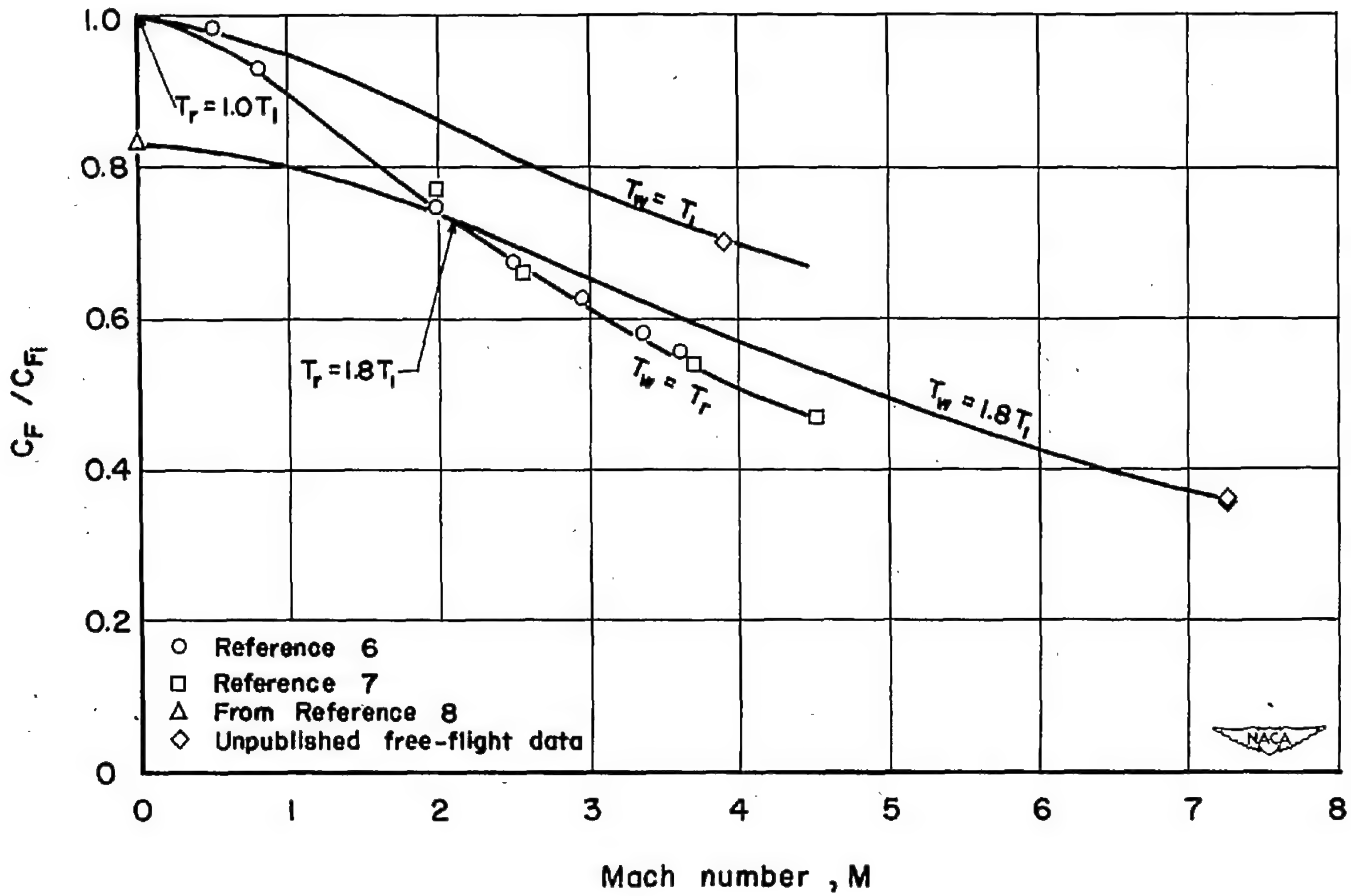

Figure 2.- Experimental data used to define the variation of turbulent skin-friction coefficient with Mach number and wall temperature ratio.

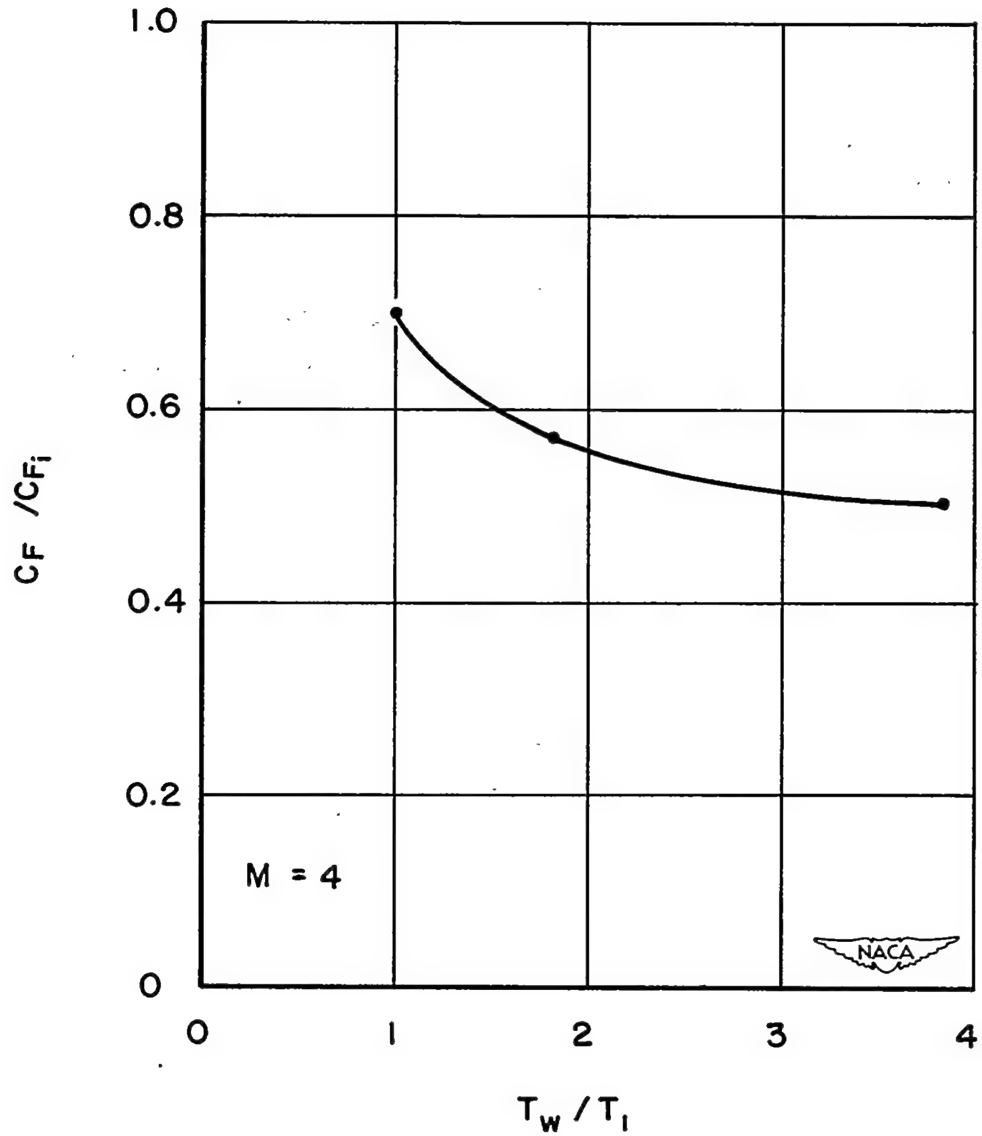

Figure 3. - Cross plot at M = 4 of curves of figure 2.

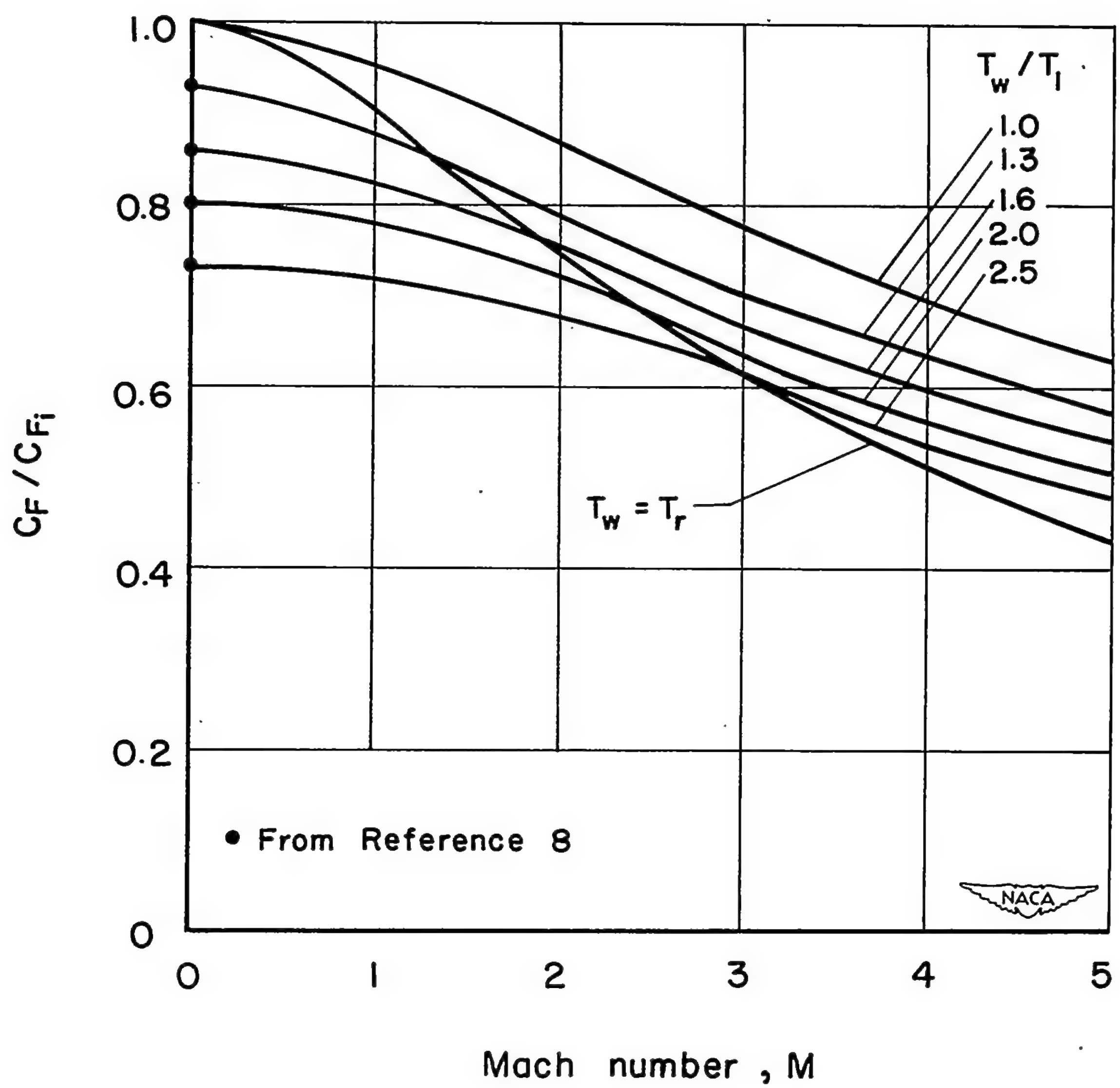

Figure 4 .- Interpolated variation of turbulent skin-friction coefficient with Mach number and wall temperature ratio.

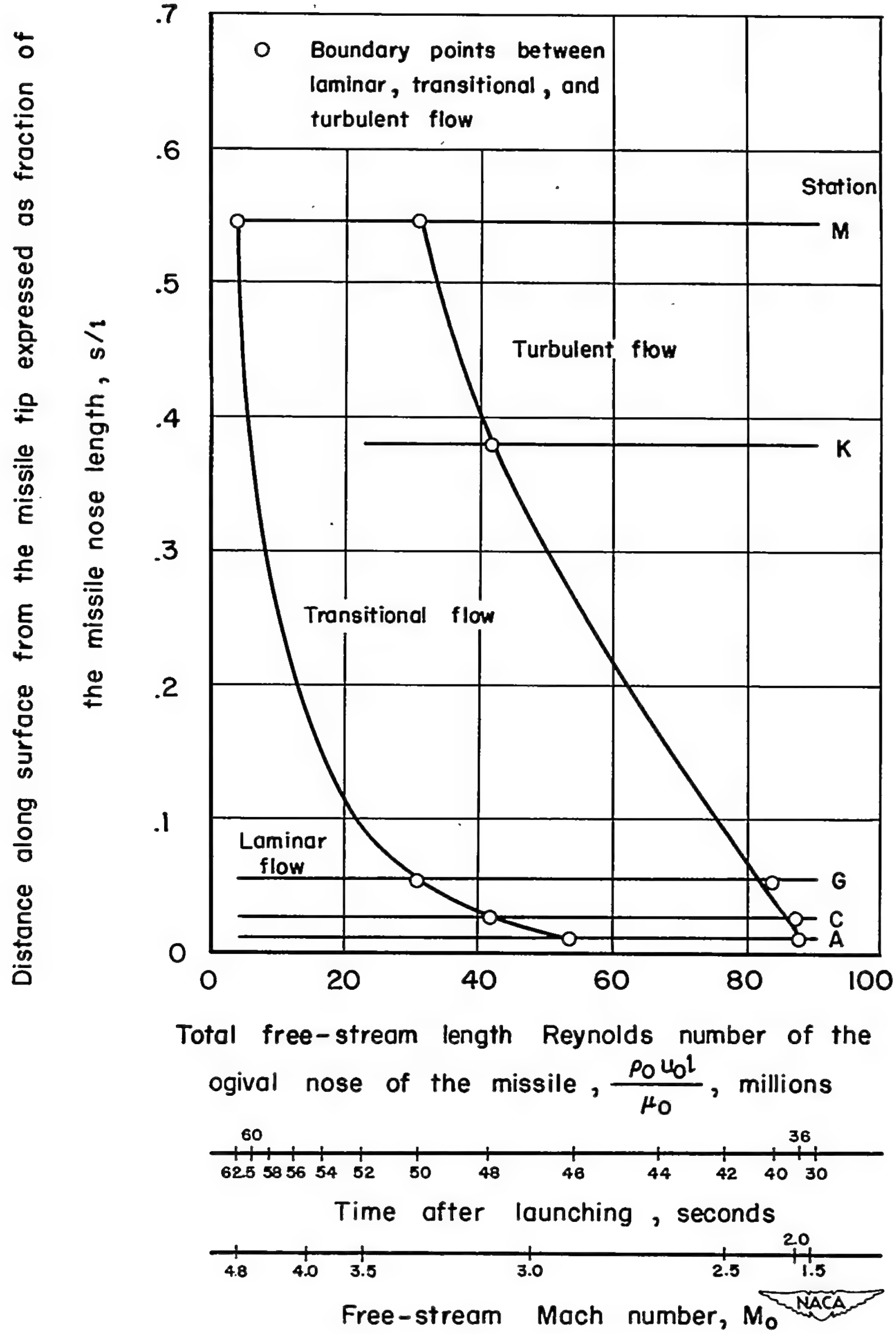

Figure 5.- The transition-history of the Fischer-Norris V-2 flight.

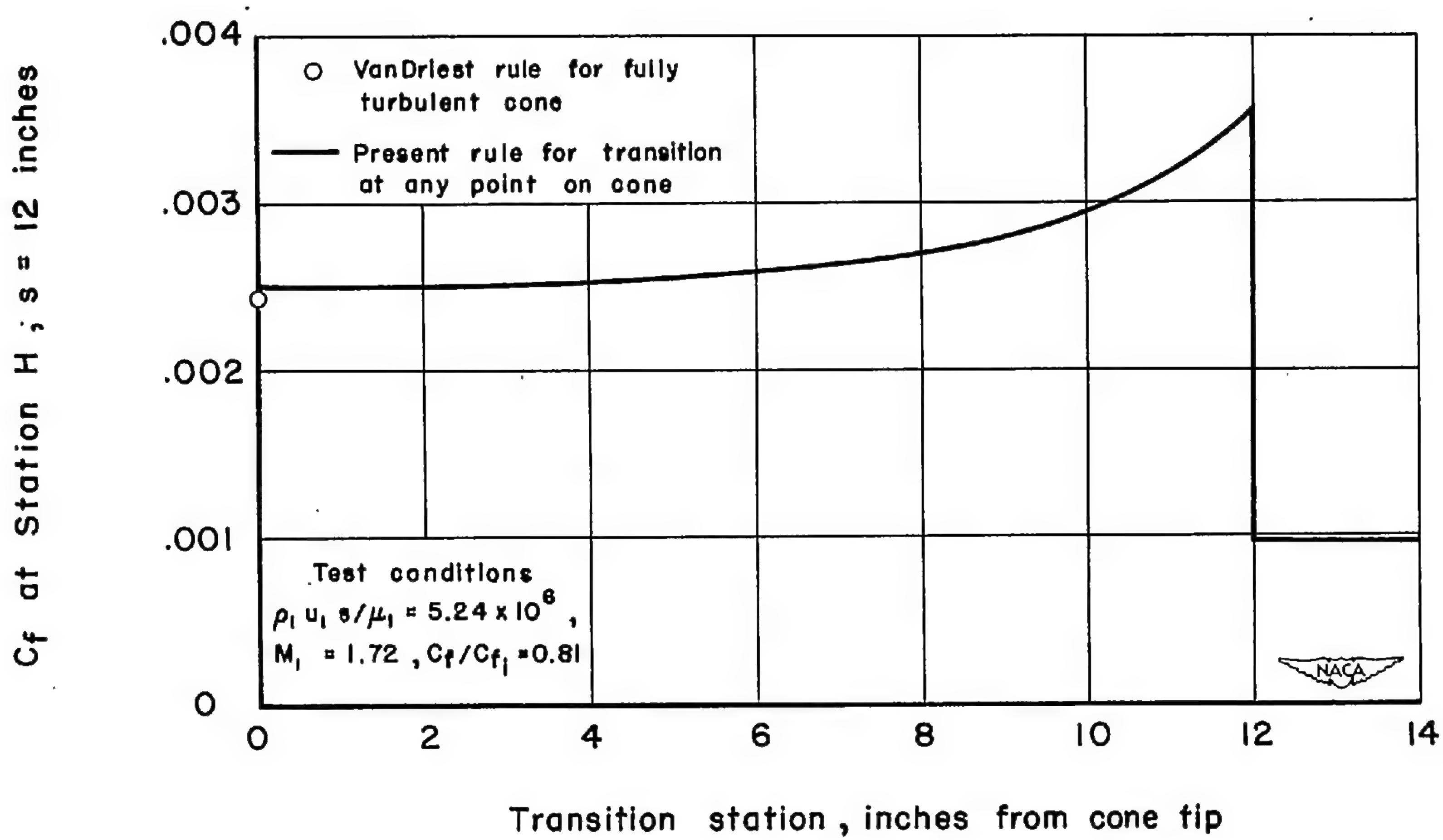

Figure 6 .- The calculated local skin-friction coefficient at Station H on the V-2 nose cone as a function of transition point.

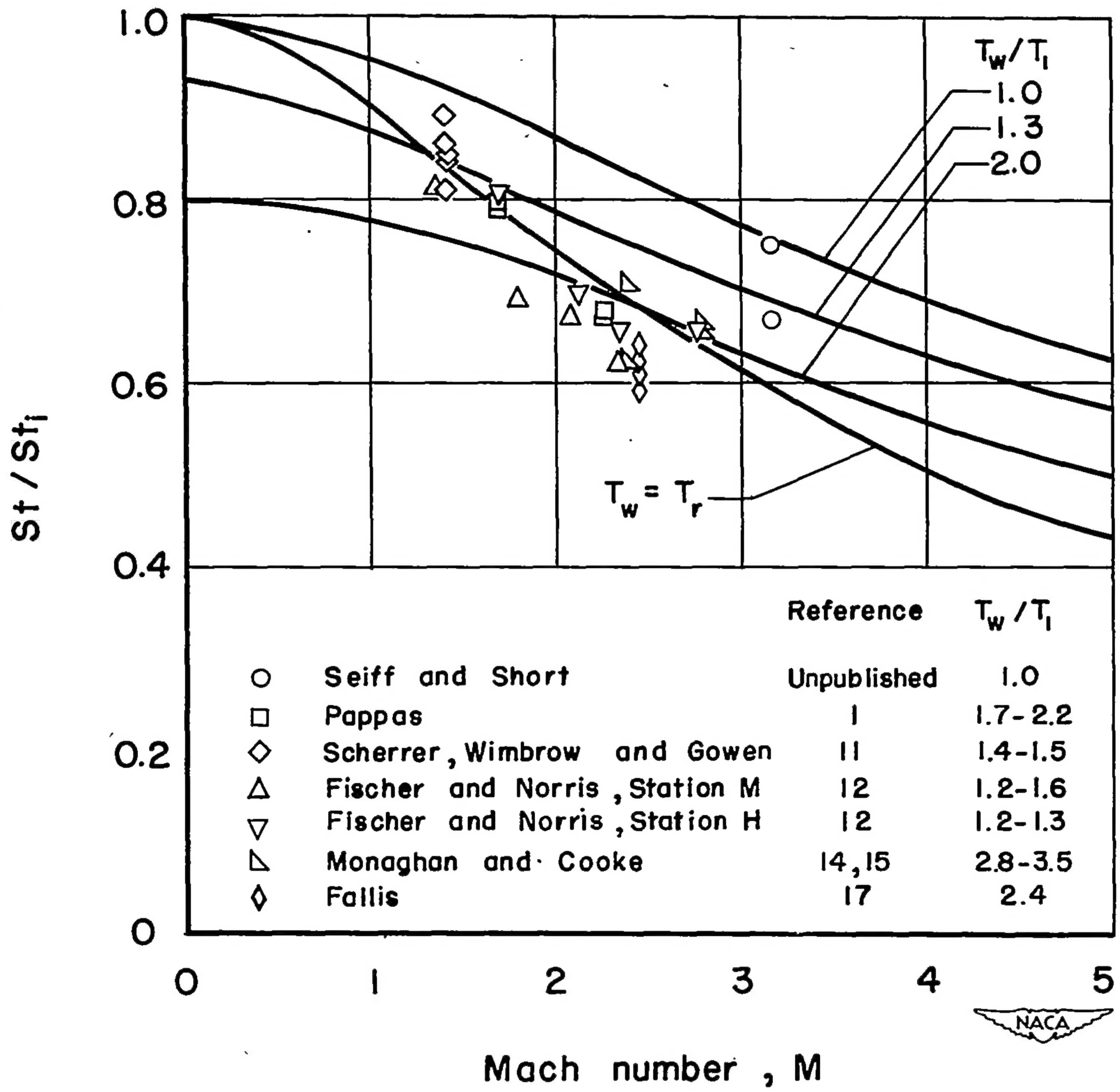

Figure 7. - The experimental data on Stanton number as affected by Mach number and wall temperature ratio, compared to the skin-friction curves of Figure 4.

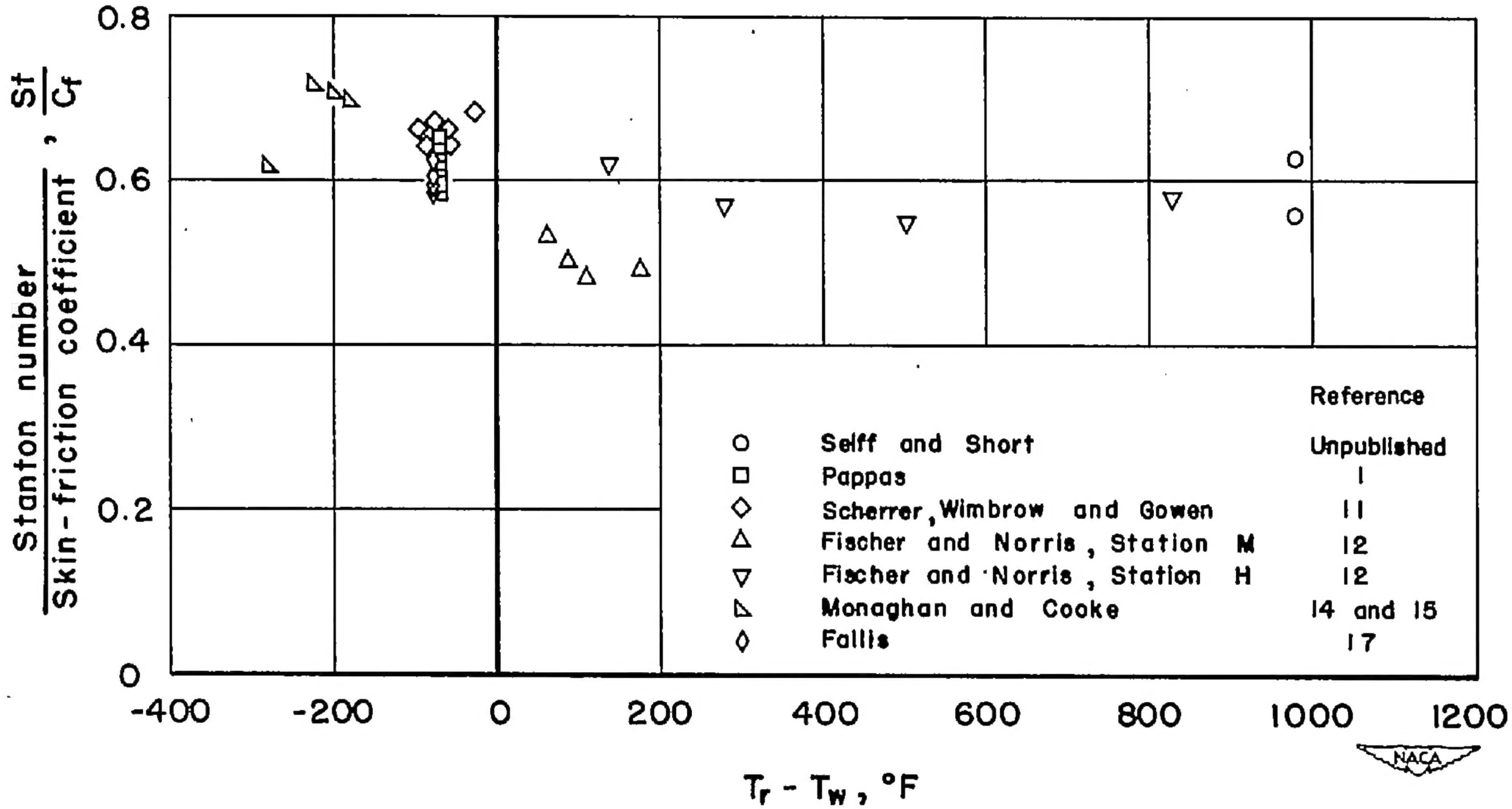

Figure 10.- The heat-transfer data plotted against the temperature difference, $T_r - T_w$, causing heat transfer.

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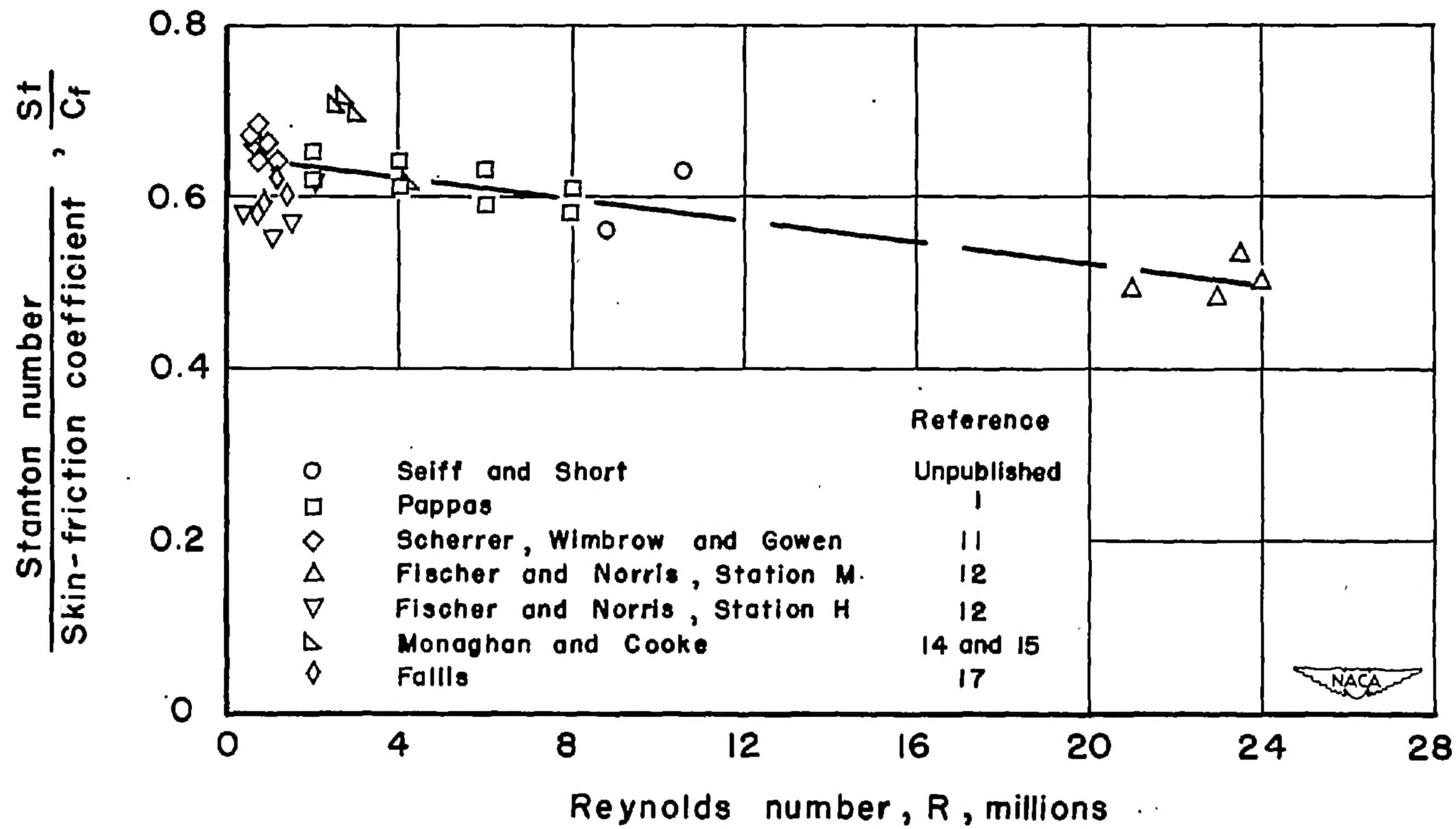

Figure 9 .- The heat-transfer data in the form St/C_f, plotted against Reynolds number.

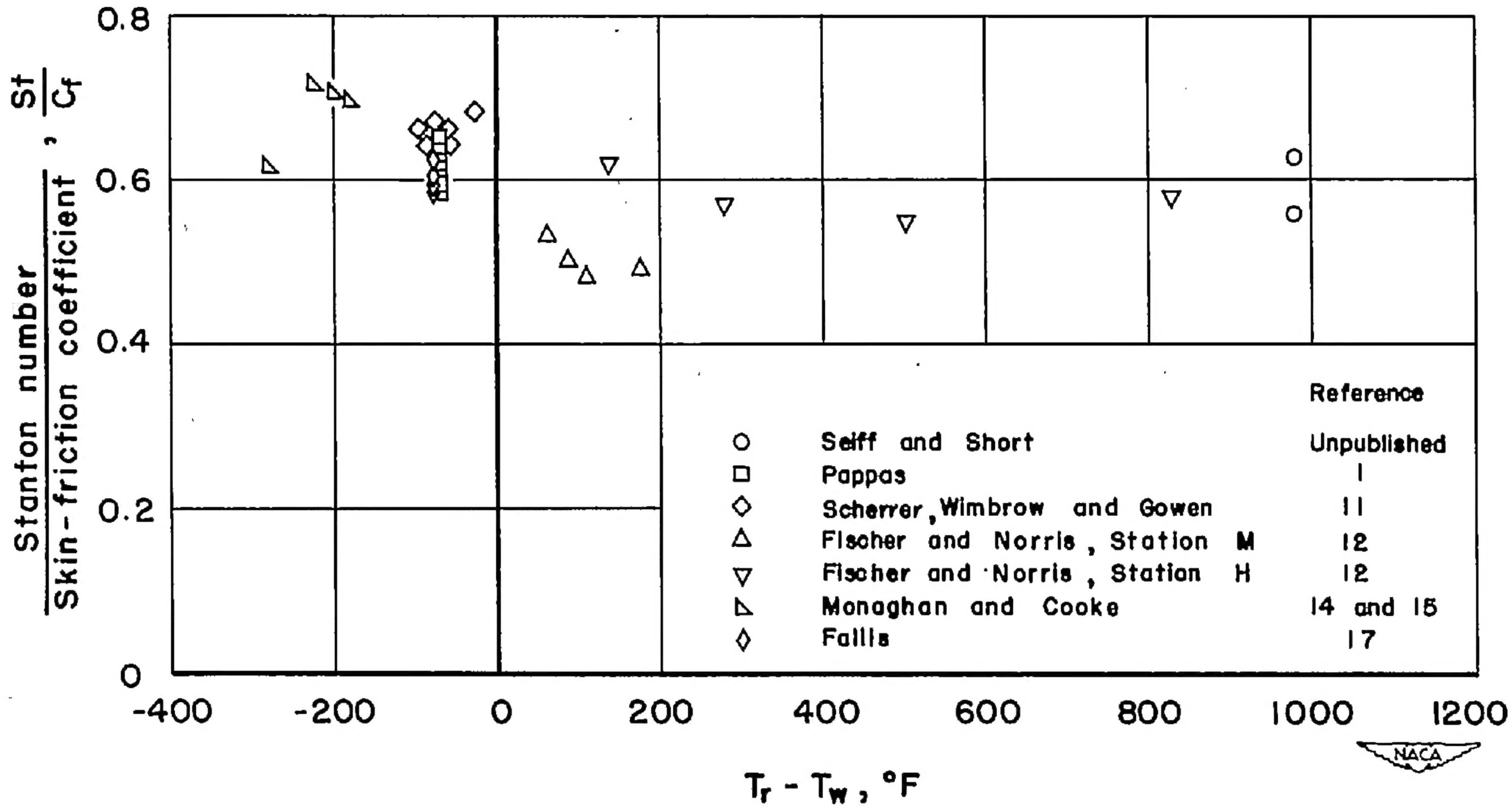

Figure 10.- The heat-transfer data plotted against the temperature difference, $T_r - T_w$, causing heat transfer.

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